

**THE NEW FIELD OF SYSTEMS ALLOMETRY: DISCOVERY OF EMPIRICAL EVIDENCE FOR INVARIANT PROPORTIONS ACROSS DIVERSE SYSTEMS**

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**ABSTRACT:** This paper presents the early results of the new field of systems allometry which studies proportional regularities that are invariant across widely different natural systems. Empirical data on 15 Newtonian and 5 information-based parameters from the refereed journals of the conventional sciences have been collected into a massive relational data bases and statistically analyzed for transdisciplinary isomorphies. Two case studies of invariant proportions are presented across levels of systems organization and entities as diverse as molecules, organelles, cells tissues/organs, organisms, communities, and ecosystems. We have found that the means of lifecycles and mass of these diverse systems are invariant according to the formula,  $y = x^5$  to the 95% significance level. We have also found that the means of linear dimensions and mass from molecules to ecosystems vary as  $y = x^2$ . These and other correlations under study are significant because they demonstrate: (i) isomorphic relations can be proven across discontinuous, emergent levels; (ii) "levels of organization" may not be just mental constructs, but real entities in nature obeying natural constraints; (iii) the hypotheses of systems science, and specifically systems allometry, generate unique and testable results not observable in the conventional sciences.

**1. The Hypotheses of Systems Allometry and Their Relation To Allometry In The Conventional Sciences and Engineering**

Most of the questions or hypotheses typical of the conventional sciences are restricted to a very tiny number of scalar levels. Questions in chemistry are restricted to chemical systems, in cell biology to cellular systems, etc. In contrast, hypotheses in systems allometry require study of empirical relations across a much wider range of scalar levels. In this study, for example, the hypotheses tested involve a range of sizes covering fully 21 orders of magnitude ( $10^{21}$ ). This feature clearly distinguishes systems allometry from allometry in the conventional sciences.

A previous paper in this series (Troncale, 1986) argued that classical bio-allometry (over 50 years old) is already a systems-level study and consequently possesses several similarities to the new field of systems

allometry. Bio-allometry attempts to measure interrelationships among measurables of biosystems which may be described as a biosystems level phenomenon. Still there are differences also between systems allometry and bioallometry. The correlations presented here will be extended to tests of physical, astronomical, and geological systems, as well as to any potential measurables of social systems in the near future. These extensions will go far beyond the boundaries of bioallometry. Further, the parameter sets compared in these tests also extend beyond those normally used in bioallometry. Finally, systems allometry is focused on answering questions pertinent to systems organization, systems stability and other systems characteristics not normally considered in bioallometry. In fact, the results of systems allometry may be used in the future to help specialists in bioallometry understand why they find the correlations they do in biological systems. Despite a renaissance of activity in classical bioallometry, biologists are still not sure why the correlations they find are required in biosystems. A wider systems understanding of their role in systems function would be one contribution of a new systems science to this conventional, well-established science.

Some of the interesting questions posed across normal disciplinary lines by the new field of systems allometry are the following. Are there linkages across what normal science considers truly different and discontinuous objects in nature? Are these regularities expressible in mathematical formulae and provable by empirical studies? How are these regularities related to systems function, in other words how do they determine systems form? Are the regularities related to systems limits that are not related to the local scalar level at all, but rather to some overall limits of systems form not yet sensed, or suspected by the conventional disciplines? Since the correlations shown by systems allometry are not necessarily cause-and-effect, do the regularities demonstrated indicate the result of systems-level networks of cause-and-effect otherwise unapproachable by the conventional disciplines? Strong answers to any of these questions would be evidence for the utility of systems science as a meso-science extending the conventional sciences (Troncale, 1988).

## 2. Methodology

A series of past papers (Troncale, 1981, 1982) describe the ongoing project of building a relational data base using dBase III which records more than 25 descriptor fields for empirical measurements collected from the refereed journals of most scientific specialties. Exact quotes, full bibliographic references, and extensive taxonomic descriptors insure the tight coupling of the data base to traceable physical experiments. For completeness, data continues to be added to the data base on all natural levels from sub-sub-atomic particles through atoms, up to astronomical systems, through the numerous levels of biological and geological organization, even to societal levels. Data has been entered on newtonian parameters such as linear measurements, mass, interaction distance, interaction energy, interaction time, numbers per level, generation time



and energy, lifespans, fecundity, and phase states. Additional data is being sought on informational parameters such as total information content per entity, minimum generational information, information flow rate, and total information complexity. Clustering analysis applied to the data has been used to try to determine the non-anthropomorphic levels of hierarchical organization in natural systems. In this attempt each parameter set has been characterized statistically for each proposed level in regard to mean, standard deviation, standard error, range and min-max values, normal Poisson distribution, sample fitness, etc. The existence of this data base also allows for a search of correlations among and between various pairwise comparisons of parameters across levels which forms the basis for systems allometry (as originally suggested by Troncale, 1982). The data base is resident on Kaypro, IBM-XT, and Cyber computers at the Institute.

The many individual data entries for any one parameter are converted to a common metric so that data can be compared across levels. Using standard statistical packages, the population of data known for a parameter for a level is converted into a mean value to represent that level of organization, and then the means are compared for two parameter sets across several levels using standard statistical packages that draw a best-fit regression line. The Central Limit Theorem supports use of the mean to represent the level in this manner. In essence, this procedure is comparing "levels of organization" for correlations, as "levels of organization" and not as individual entities, which is the intent of a systems-level allometric study.

### 3. Empirical Evidence For Invariant Proportions Across Diverse Systems

For the data that follows, a more complete characterization of the data set for each parameter and level may be found in Troncale (1982). The regression line graphs shown here were produced on Lotus software and so reject the correlation at a 95% level in a manner similar to the natural sciences. That these studies are truly transdisciplinary is indicated by the observation that in one of the graphs mass dimensions range across 35 log scales - clearly beyond the domain of any one discipline, although within the domain of the cohort of disciplines represented by living systems.

#### 3.1 THE LINEAR DIMENSIONS OF DIVERSE SYSTEMS OCCUR IN INVARIANT PROPORTION TO THEIR MASS

In the process of origins of various levels of biological systems, there is observed a discontinuity of emergent qualities. Although all subsequent levels of biological organization are composed of biopolymers, it is clear that ecosystems or organisms, or cells for that matter, are not the same things as molecules. Indeed, the times of origin of each level are separated in time and space in large amounts as biopolymers originated circa  $4.0 \times 10^9$  years ago, cells circa  $3.5 \times 10^9$  years ago, organelles circa  $2.5 \times 10^9$  years ago, organisms at the same time as cells since it

is such a loose, unscientific term, and communities and ecosystems originating in two waves (unicellular versus multicellular). Is it reasonable or conventional to hypothesize that levels of organization so separated in emergent qualities, mechanisms, and time of origin should exhibit constant relationships between their characteristic parameters? If they do, and such a relationship is empirical, it would become an argument for the importance of a generalized systems form as yet unrecognized.

Table One shows the means of the data sets for several levels of living system from molecules to ecosystems for two parameters, linear dimensions and mass.

TABLE I

Level	Mean Value Mass (Kgs)...N #	Mean Value Linear (Met.)...N #
Molecules	$2.44 \times 10^{-22}$ .....73	$1.10 \times 10^{-8}$ .....18
Organelles	$7.89 \times 10^{-21}$ .....10	$2.73 \times 10^{-7}$ .....266
Cells	$3.98 \times 10^{-14}$ .....6	$1.77 \times 10^{-5}$ .....272
Tiss/Organs	$4.31 \times 10^{-1}$ .....29	$6.08 \times 10^{-3}$ .....94
Organisms	$3.86 \times 10$ .....11	$3.79 \times 10^{-1}$ .....126
Communities	$1.48 \times 10$ .....4	$1.21 \times 10^5$ .....7
Ecosystems	$9.81 \times 10^{12}$ .....14	

Clearly, some of the parameter sets are yet too small to achieve a valid standard error indicating sample fitness. It will be interesting to see if the means used in the following study change much with added data. The data set used to generate the regression line in Figure One is restricted to the use of six of the seven levels represented above, and uses 147 measurements from the refereed literature on the mass characteristics of these levels, and 783 measurements from the literature on linear characteristics for linear dimensions. The graph shown represents the comparison of the logs of 12 means constructed from 930 measurements taken from specialty journals of the conventional sciences.

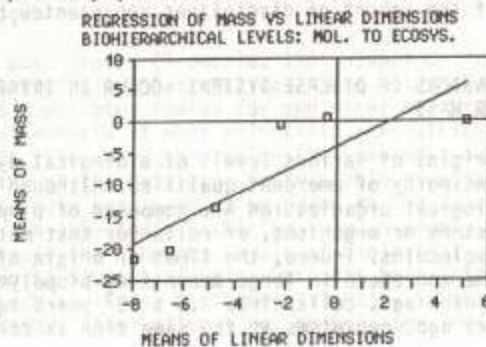




Figure One shows the regression of mass versus linear dimension for the following biohierarchical levels reading the points left to right; molecular, organellar, cellular, tissue/organ, organism, eco-community. The "best fit" regression line drawn through the points exhibits the following regression characteristics (given the 6 observation points and the 4 degrees of freedom): the Constant (y intercept) = -3.76; the Standard Error of Y = 5.92; R Squared = 0.75; the Correlation Coefficient = 0.86; the X Coefficient = 1.91; and the Standard Error of Coef. = 0.557.

The correlation coefficient is greater than the 0.811 required for significance at the 95% confidence level. About 75 % of the data is explained by the regression line. The regression line itself indicates that there is a significant correlation between the masses characteristic of a hierarchical level and their linear dimensions, NO MATTER WHAT LEVEL ONE EXAMINES, ACROSS ALL LEVELS, AND DESPITE THEIR OBVIOUS DIFFERENCES OF FORM AND MECHANISM, AS WELL AS TIMES OF ORIGIN hundreds of millions of years apart.

This regression line can be expressed in an allometric equation (see Troncale, 1986) using the anti-log of the Y intercept as the constant (or scale factor) and the calculated X coefficient as the slope, and recognizing that the former is so small a number and rounding 1.9 to 2 we get

$$Y = X^2$$

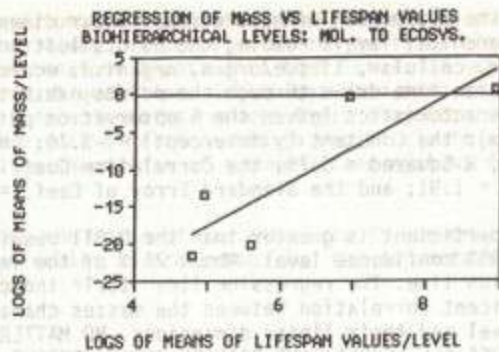
or, the mass of a hierarchical level of organization in all biosystems so far studied varies directly as the square of its linear dimensions. Because this is an empirically based correlation on the level of "level of organization" it is in the domain of systems science and may allow a new era of testable systems science theory that describes the "right" or workable sizes or proportions of systems forms in certain definable systems domains.

### 3.2 THE LIFESPANS OF DIVERSE SYSTEMS OCCUR IN INVARIANT PROPORTION TO THEIR MASSES

In this next pairwise comparison, five levels are tested using 502 empirical measurements. Again about 75 % of the data are explained by the regression line shown in Figure Two. However, the correlation coefficient of 0.862 is just below the 0.878 required for the 95 % confidence level test. Examination of each level compared indicates that the molecular and organellar levels have restrictively small numbers of data in their data sets. Inclusion of additional data may improve the correlation coefficient.

The "best fit" regression line shown in Figure Two on the regression of mass versus lifespan values from molecules to organisms (reading the points again from left to right) has a very interesting allometric equation. The scale factor in this instance is so small (1.27 preceded by 44 zero's) that it can be ignored. The resulting equation reads

$$Y = X^5$$



This very interesting equation for Figure Two (above) remember explains 75 % of the data shown and collected in the means. The confidence test is based to a significant degree on the degrees of freedom which are restricted in this case due to exclusion of ecological data which is available in the paleoecological literature not yet explored. It is my assumption that inclusions of these two additional levels will raise the correlation coefficient to the amount needed to pass the test for significance. If this is the case, then we again have a regular relationship between mass and another important systems characteristic - lifespans - where the latter is directly proportional to the fifth power of the former. This relationship holds true across the diverse nature of each bio-organizational level and has the same impact as the last mentioned correlation.

Will the same allometric equation retain this level of near significance at the 95 % level when data from physical (atomic) and astronomical levels of organization are included? Retention of such a regularity across even more dramatically different cohorts of levels as across living and non-living systems would be a potent result for systems science, clearly unsuspected in the conventional disciplines.

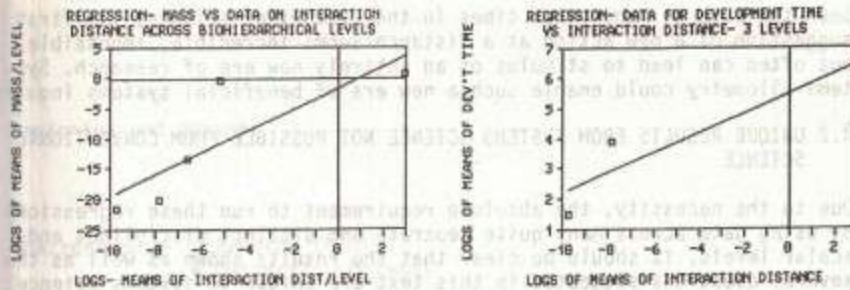
### 3.3 OTHER POTENTIAL PROPORTIONAL SIMILARITIES UNDER STUDY

Figure Three shows a regression of means of mass values against means of interaction distance across biohierarchical levels. Five levels and 356 empirical measurements are represented, 67 % of the data is explained by the "best fit" regression line, although the correlation coefficient is 0.82 and falls below the 0.88 required for 95 % confidence in the correlation. With the present data set the allometric equation for the regression line shown is

$$y = .03 x^{1.78}$$

Interaction distance is defined as "the maximum distance across which





two entities at a level can still interact successfully given the type of interaction typical for that level." Figure Four on the right above, is included here to indicate that all parameters need not be compared with mass. The regression line here suffers from a current paucity of data matching development time for a level of system with its interaction distance. Fully 85 % of the data is explained by this line, and if the data follows the same pattern after adding three more bio-levels, and the correlation coefficient remains the same, this category of proportion would pass the 99 % confidence level test.

In summary, these last two attempts indicate that there is much more of interest to pursue in cross-disciplinary, trans-scalar isomorphies within the field of systems allometry.

#### 4. Conclusions and Significance

These statistical analyses are still primitive and preliminary. It is hoped that they provide a first glimpse or demonstration of how the new field of systems allometry might help systems science establish itself as a more scientific, less purely theoretical pursuit. Further, it would link the results of systems science to the empirical findings of the conventional disciplines increasing, perhaps, their respect for its conclusions, but also demanding that they look at the physical world in a radically different, more systematic way than before.

##### 4.1 LEVELS OF ORGANIZATION AS REAL, NATURAL ENTITIES

Why would such disparate entities as those just studied consistently exhibit the same mathematical proportions when viewed as a high degree of abstraction and synthesis? The results, if expanded, might indicate that levels of organization are themselves not just convenient, taxonomic collections created by the human mind, but rather are very real things that nature constrains as they emerge at different times in different places and combinations. But, then, the very significant question emerges which asks what mediates this synthetic action at a distance. As has

been the case so many other times in the history of science, the first suggestion of a new action at a distance seems incredible, impossible, but often can lead to stimulus of an entirely new era of research. Systems allometry could enable such a new era of beneficial systems inquiry.

#### 4.2 UNIQUE RESULTS FROM SYSTEMS SCIENCE NOT POSSIBLE FROM CONVENTIONAL SCIENCE

Due to the necessity, the absolute requirement to run these regressions by using data across many quite separate and distinct disciplines and scalar levels, it should be clear that the results shown as well as the several questions suggested in this text are unique to systems science. They could not be conceived in the regular sciences nor asked in normal science since they fall between the cracks between the sciences. Further, the understanding that emerges from the correlations, and the sense of systems function that are required to explain the correlations may return to the conventional sciences a new insight they could not have attained by themselves (and return the favor we gain from them in borrowing their valuable empirical output). The puzzle that faces biologists who demonstrate, but cannot explain their bioallometries may be serviced by advances in systems science. Until such a time, when systems science yields results of interest to other sciences, it will not be regarded seriously. See Troncale (1987) for predicted formulae from this study.

#### 4.3 PRESERVATION OF SIMILARITIES ACROSS DISCONTINUITY AND EMERGENCE

On its own level of systems isomorphies, systems allometry may contribute to systems science by re-opening questions about the interrelationships between established isomorphies and theories regarding catastrophe theory, discontinuity, and theories of emergence, as well as certain results in hierarchy theory. All stress the "gaps" between levels or states of systems. Yet, how powerful and real are these gaps if certain regularities survive across them? What is the significance to chains of origins typical of the metahierarchy (Troncale, 1982), and of concepts of origin like autopoiesis and self-organization, if these regularities persist across most such cases so far studied? It should be clear that systems allometry may contribute to a better empirical refinement of these areas of systems research.

#### 5. References

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